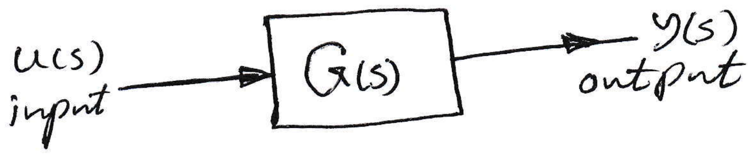
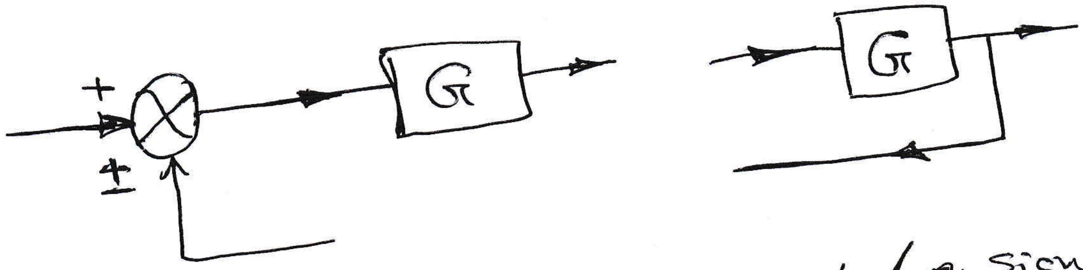
3- Block Diagram :-

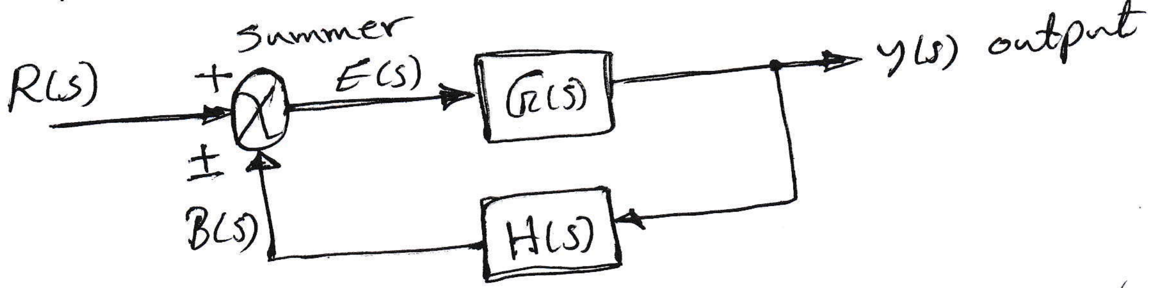
The transfer function of any system can be represented by a block diagram as shown below.



The sum or difference of signals is indicated in the diagram by a symbol called summer, as shown below



A simple feedback with the associated signals



$G(s) = \dfrac{Y(s)}{E(s)}$: Forward path transfer function, or plant transfer function.

$H(s) = \dfrac{B(s)}{E(s)}$: Transfer function of the feedback elements.

Where, $R(s)$: Reference input.

$Y(s)$: output variable.

$B(s)$: Feedback signal.

$E(s)$: Error signal.

$G(s)H(s)$: Loop transfer function.

The closed loop system can be replaced by a single block by finding the transfer function $\frac{Y(s)}{R(s)}$

$$Y(s) = G(s)E(s)$$

$$E(s) = R(s) \pm B(s)$$

$$= R(s) \pm H(s)Y(s)$$

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)]$$

$$Y(s)[1 \mp G(s)H(s)] = G(s)R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

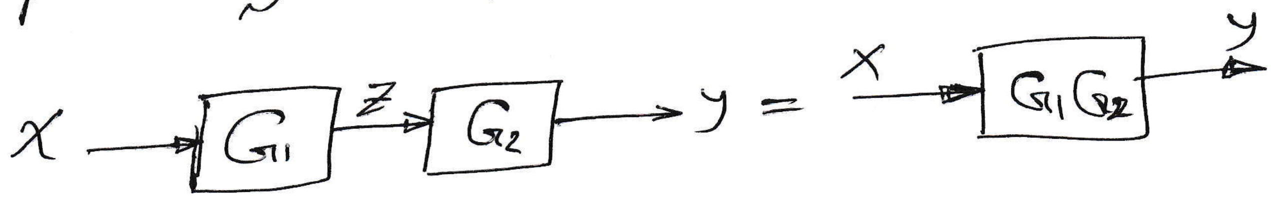
$$R(s) \longrightarrow \boxed{\frac{G(s)}{1 \mp G(s)H(s)}} \longrightarrow Y(s)$$

The most common cas of negative feedback.

$$R(s) \longrightarrow \boxed{\frac{G(s)}{1 + G(s)H(s)}} \longrightarrow Y(s)$$

3-1 Block Diagram Reduction Techniques :-

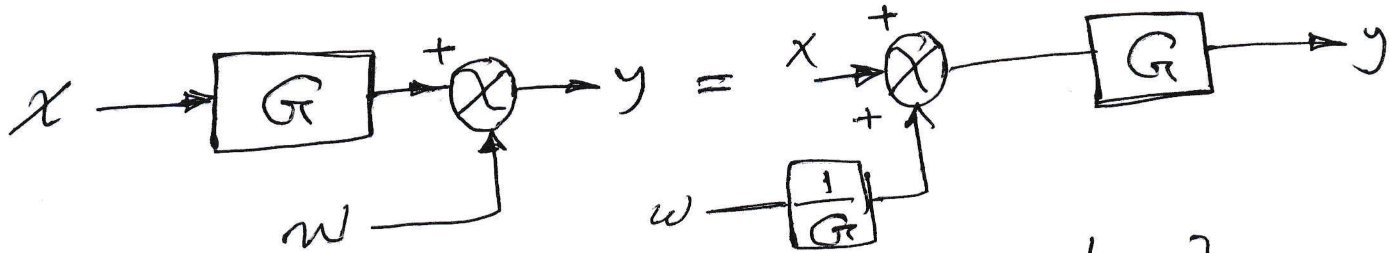
Rule 1 :- Two blocks G_1 and G_2 in Cascade can be replaced by a single block.



$$\frac{Z}{X} = G_1 \quad , \quad \frac{y}{Z} = G_2$$

$$\therefore \frac{y}{X} = \frac{Z}{X} * \frac{y}{Z} = G_1 G_2$$

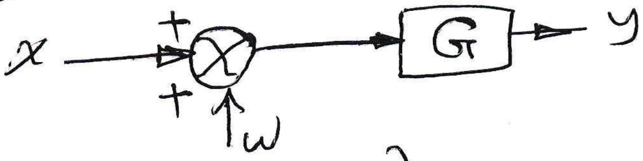
Rule 2 :- A summing point can be moved from right side of the block to left side of the block.

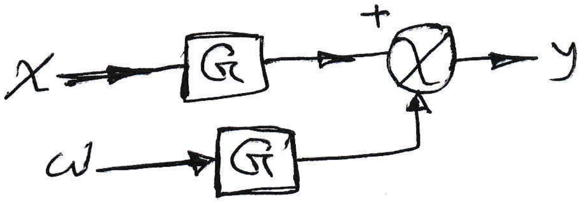


$$y = G[X] + W \qquad\qquad y = G\left[X + \frac{1}{G}[W]\right]$$

$$= G[X] + W$$

Rule 3 :- A summing point can be moved from left side of the block to right side of the block.

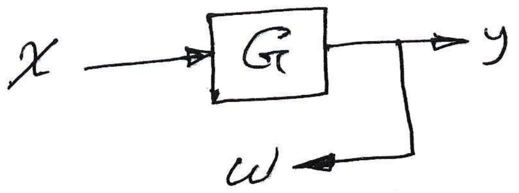


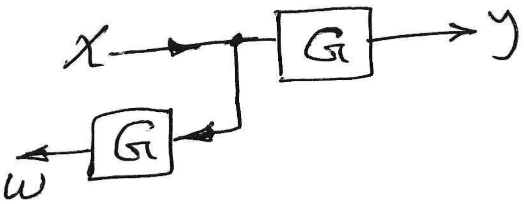


$$y = G(X+W) \qquad\qquad y = G[X] + G[W].$$

$$= G[X] + G(W)$$

Rule 4 :- A pick off point can be moved from the right side of the block to left side of the block.





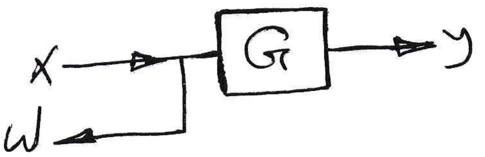
$y = G[X]$

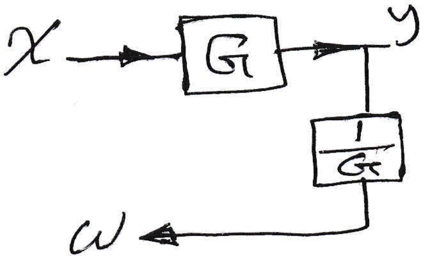
$w = y$

$y = G[X]$

$w = G[X] = y$

Rule 5 : A pick off point can be moved from left side of the block to the right side of the block.





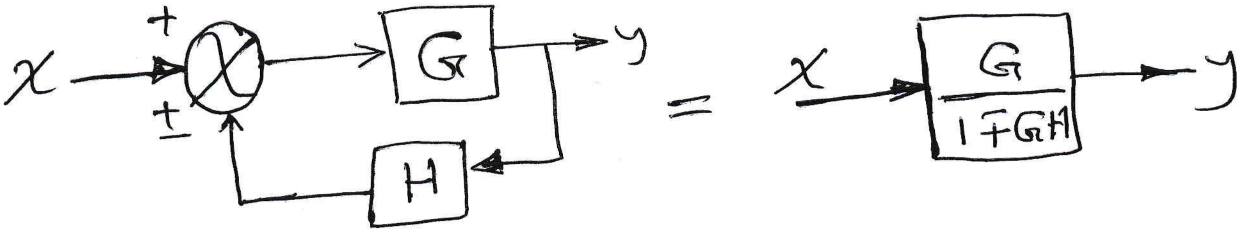
$y = G[X]$

$w = X$

$y = G[X]$

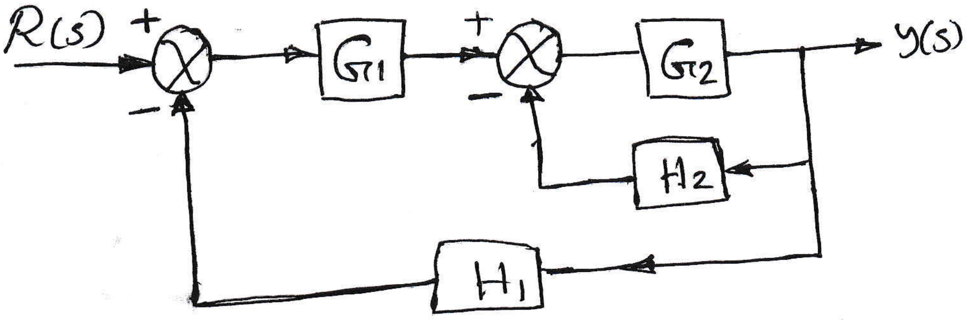
$w = \frac{1}{G}[y] = X$

Rule 6 :- A feedback loop can be replaced by a single block

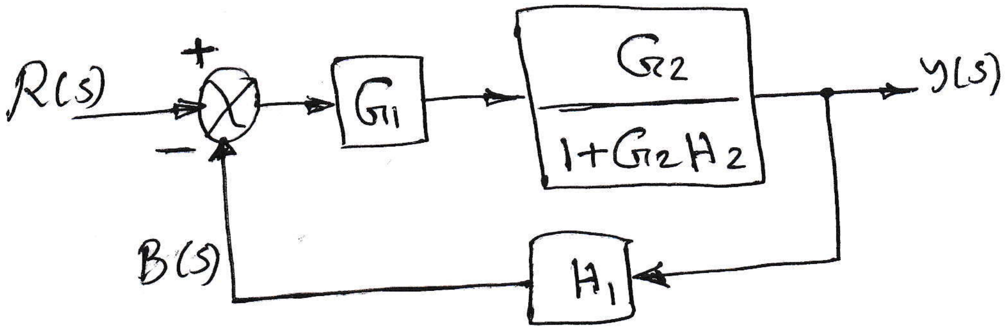


Ex

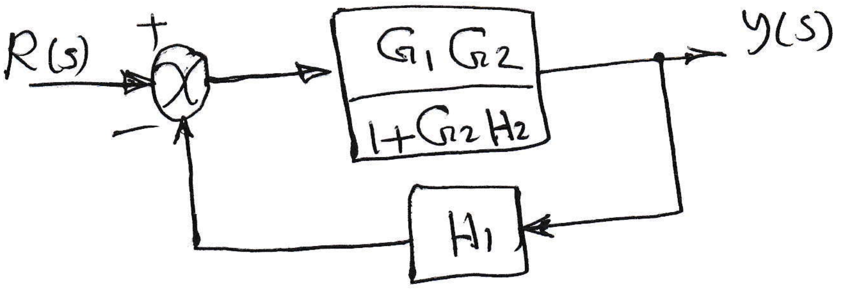
Find the overall transfer function of the system $\frac{Y(s)}{R(s)}$ in figure below, using block diagram reduction technique.



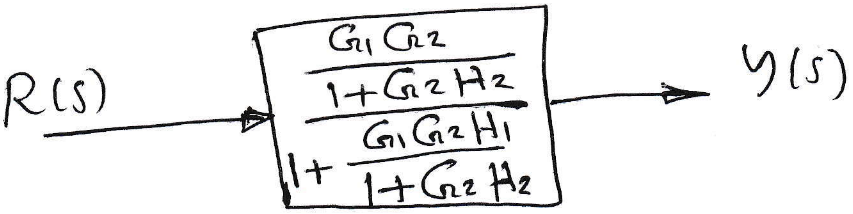
① Reducion the inter loop.



② Using forward path



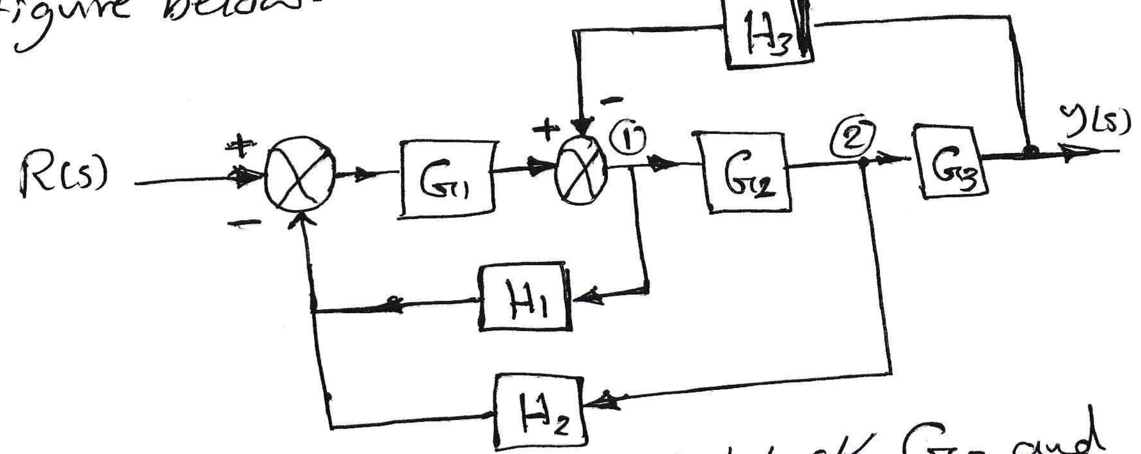
③ Reduction the final blocks



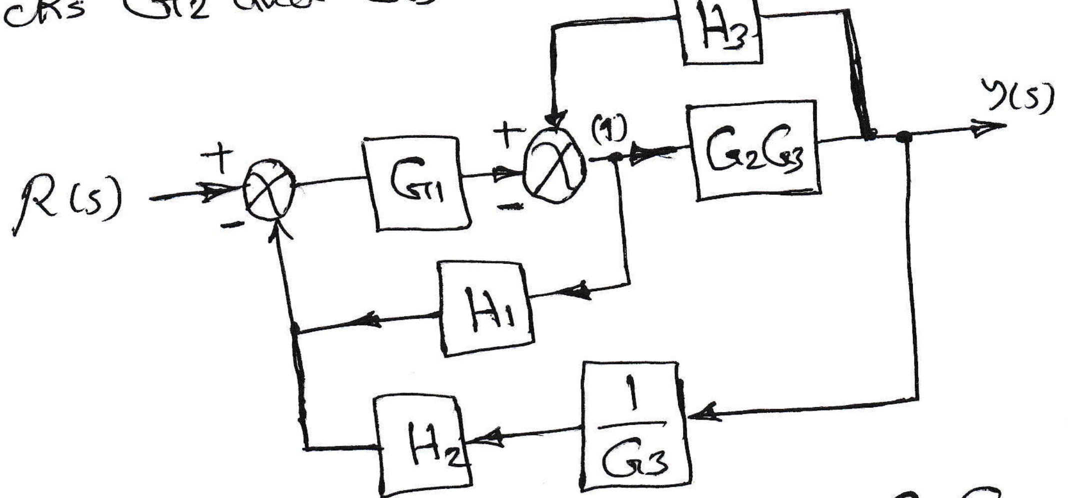
the transfer function $\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1+G_2 H_2+G_1 G_2 H_1}$

$\underline{Ex}$ obtain the overall transfer function of the system

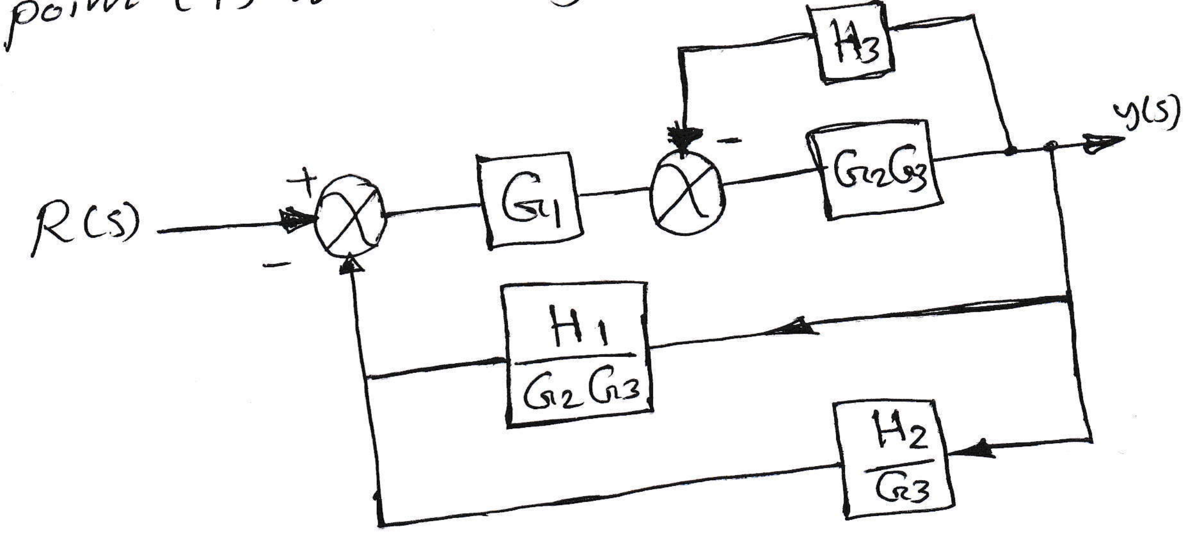
$\frac{y(s)}{R(s)}$ in figure below.



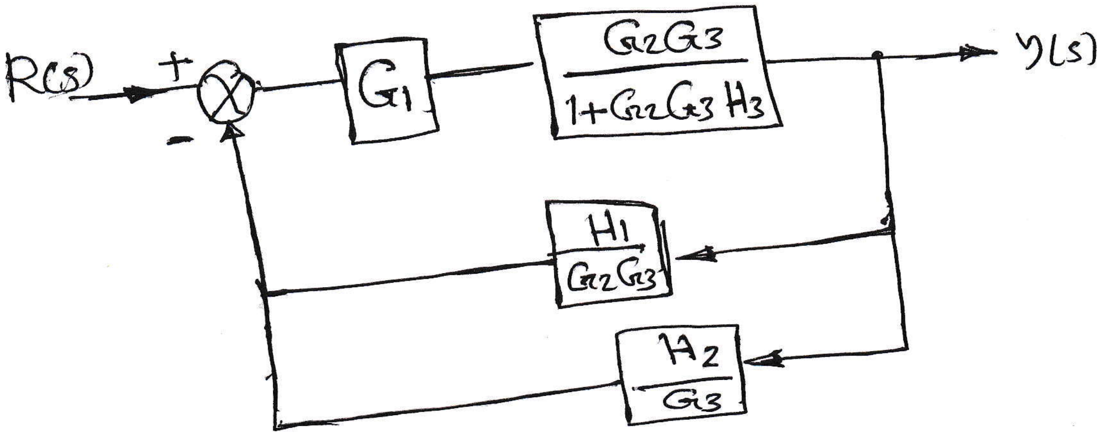
(1) moving point (2) to the right of block G_3 and Combining blocks G_2 and G_3.



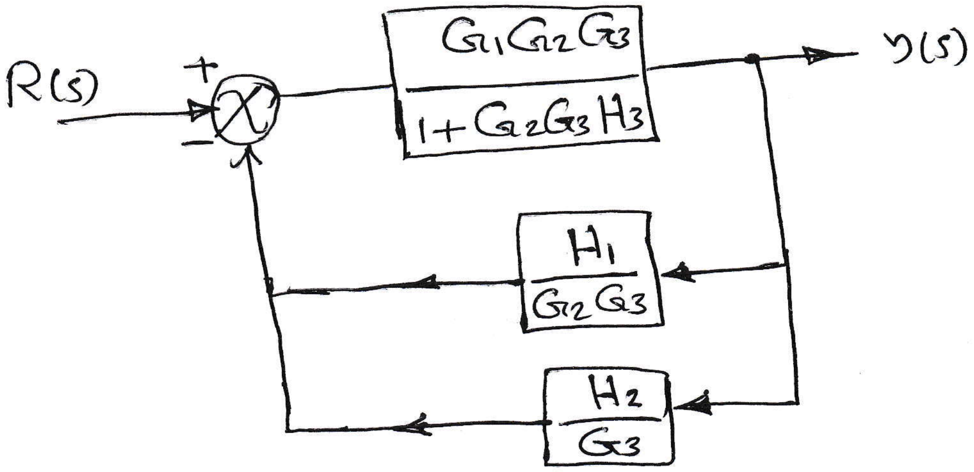
(2) moving point (1) to the right of block G_2G_3.



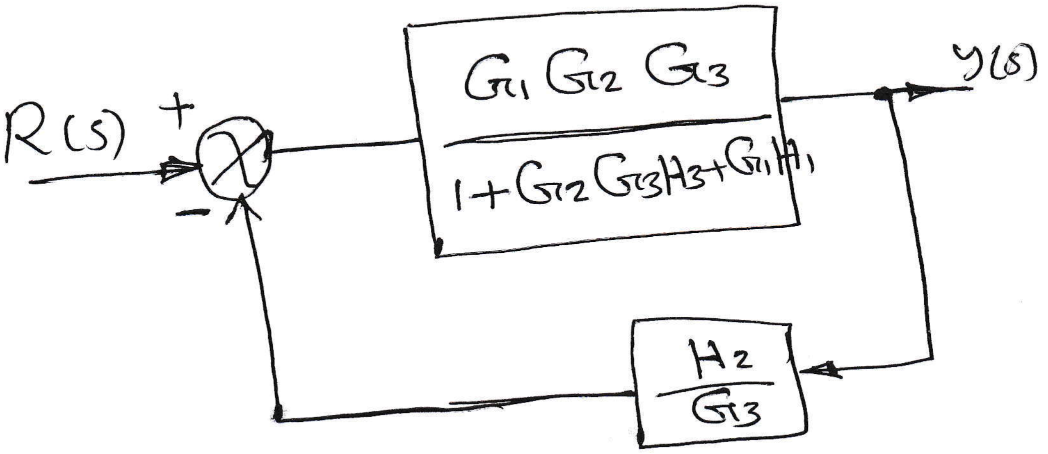
(3)



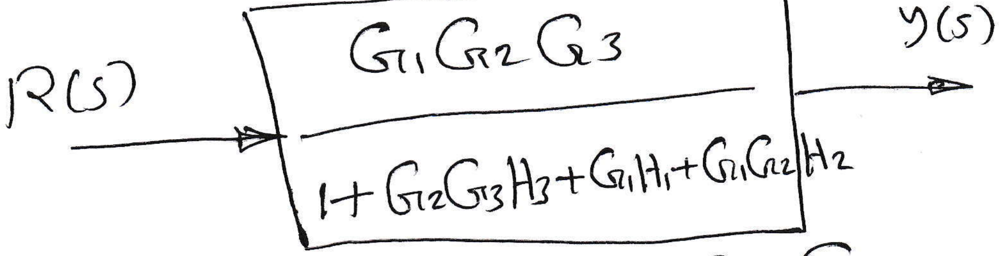
(4)



(5)

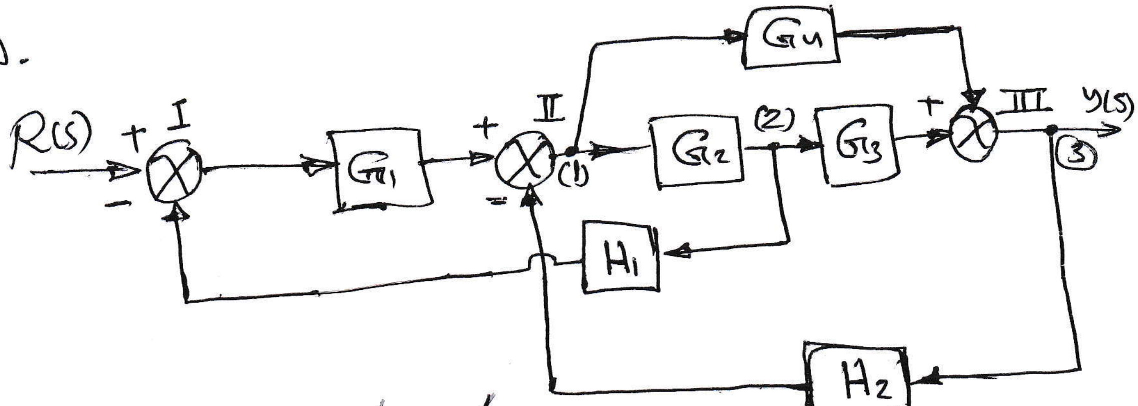


(6)

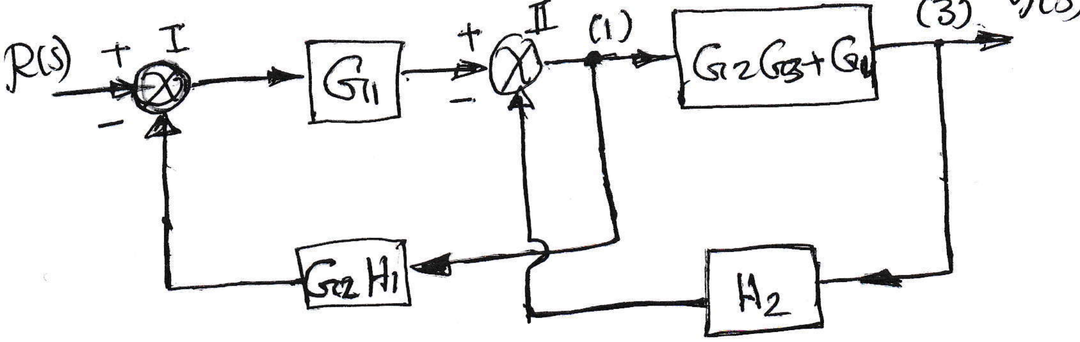


$\therefore$ Transfer function $\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2G_3H_3+G_1H_1+G_1G_2H_2}$

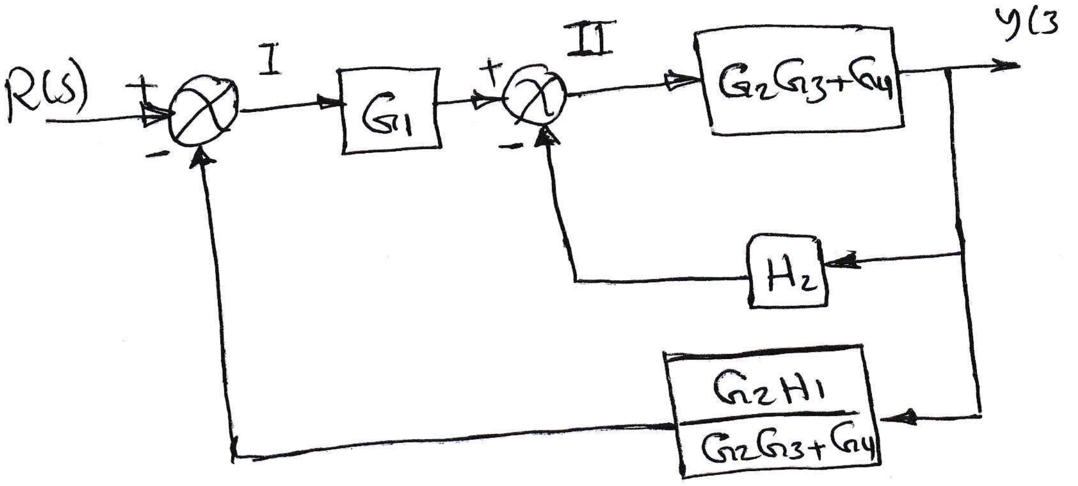
Ex Reduce the block diagram and obtain $\frac{Y(s)}{R(s)}$ in figure below.



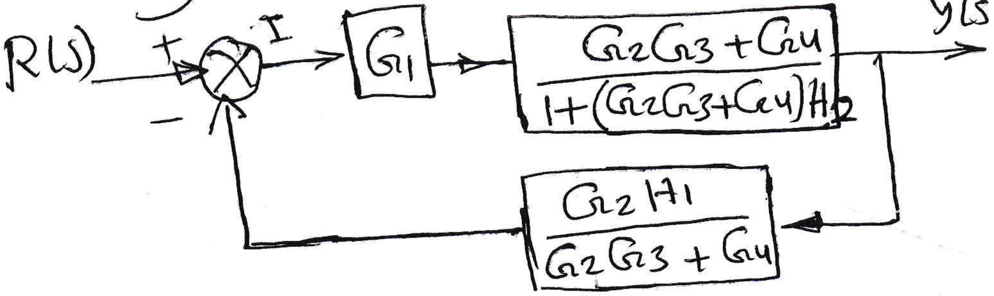
(1) Moving point (2) to (1) and get



(2) moving point (1) to (3) and get



(3) Reduction summing point II



(4) Reduction final closed Loop

$$R(s) \longrightarrow \boxed{\dfrac{\dfrac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_4H_2}}{1 + \dfrac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_4H_2} \times \dfrac{G_2H_1}{G_2G_3 + G_4}}} \longrightarrow Y(s)$$

$$\therefore \quad T.F = \frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + H_2(G_2G_3 + G_4) + H G_1G_2}$$

3-2 Signal Flow Graph Representation of Control Systems :-

Another useful way of representating a system is by a signal flow graph. Although block diagram representation of a system is a simple way of describing a system, it is rather cumbusome to use block diagram algebra and obtain its overall transfer function. A signal flow graph describes how a signal gets modified as it travels from input to output and the overall transfer function can be obtained very easily by using Mason's gain formula.

Let us define contain terms.

1- Signal flow graph

It is a graphical representation of the relationships between the variables of a system.

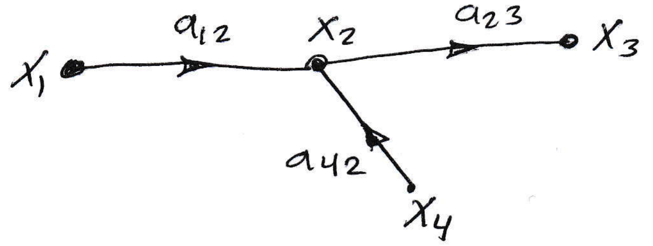
2- Node

Every variable in a system is represented by a node. The value of the variable is equal to the sum of the signals coming towards the node.

3- Branch

A signal travels along a branch from one node to another node in the direction indicated on the branch. Every branch is associated with a gain constant or transmittance.

for example $x_2 = a_{12} x_1 + a_{42} x_4$



4- Input node

It is a node at which only going branches are present. It is also called as source node x_1 is an input node, It is also called as source node.

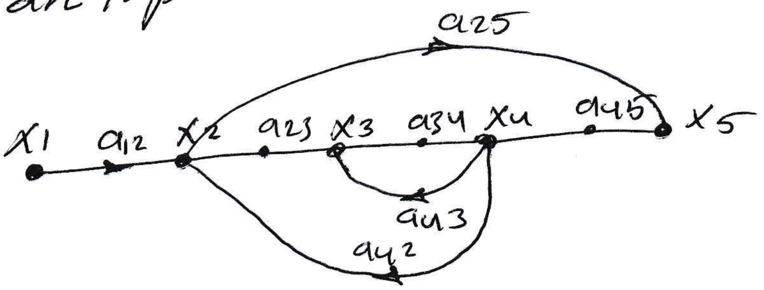


Figure (1)

5- output node

It is a node at which only incoming signals are present. X_5 is the output for Fig (1), the X_5 is output node. X_4 is the output node.

6- Path: It is the traversal from one node to another node through the branches in the direction of the branches such that no node is traversed twice.

7- Fordword path :- It is a path from input node to output nod. for example,

the forward path, $x_1 - x_2 - x_3 - x_4 - x_5$

the forward path, $x_1 - x_2 - x_5$

8- Loop :- It is a path starting and ending on the same nod, for example,

Loop, $x_3 - x_4 - x_3$

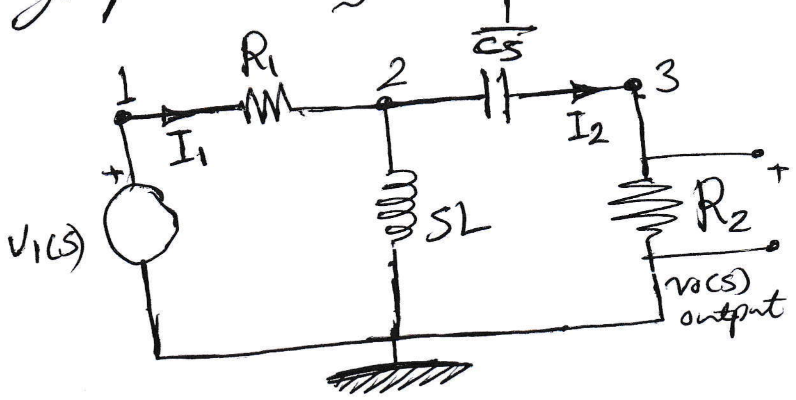
Loop, $x_2 - x_3 - x_4 - x_2$

9- Non touching loops :- Loops which have no common node, are said to be non touching loops.

10- Forward path gain :- the gain product of the branches in the forward path is called "forward path gain".

11- Loop gain :- the product of gains of branches in the loop is called as "Loop gain".

EX consider a signal flow graph for figure below.

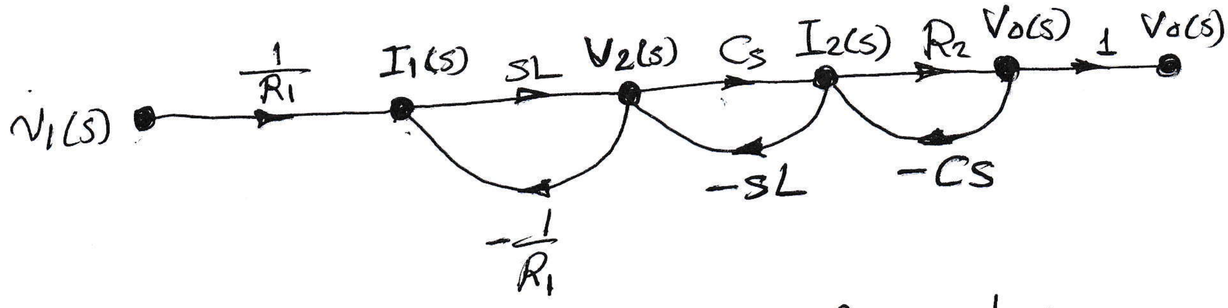


$$I_1(s) = \frac{V_1(s) - V_2(s)}{R_1}$$

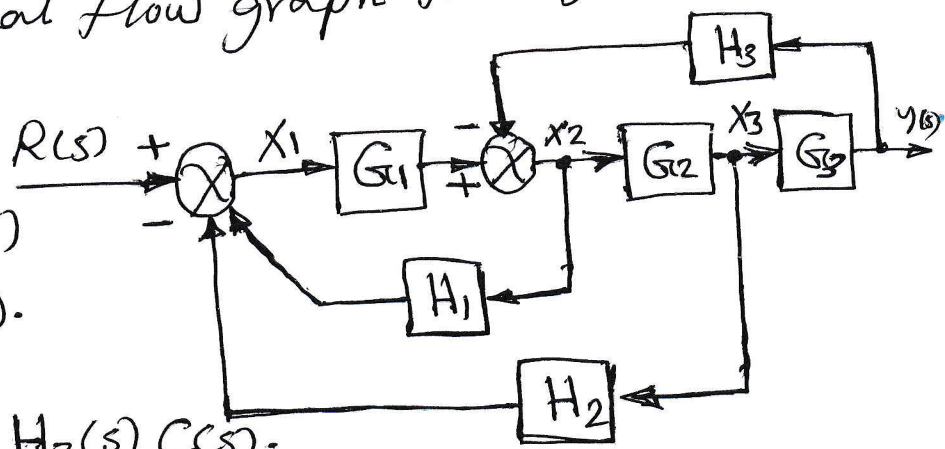
$$V_2(s) = [I_1(s) - I_2(s)]SL$$

$$I_2(s) = [V_2(s) - V_o(s)]Cs$$

$$V_o(s) = I_2(s)\, R_2$$



EX obtain the signal flow graph for figure below

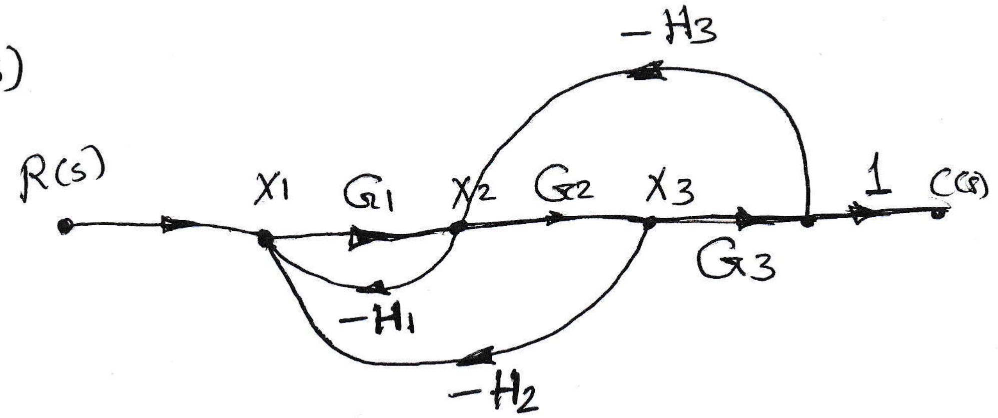


$$X_1(s) = R(s) - H_1(s)X_2(s) - H_2(s)X_3(s).$$

$$X_2(s) = G_1(s)X_1(s) - H_3(s)C(s).$$

$$X_3(s) = G_2(s)X_2(s).$$

$$y(s) = G_3(s)X_3(s)$$



3-3 Mason's Gain Formula :-

The transfer function (gain) of the given signal flow graph can be easily obtained by using Mason's gain formula.

Mason's gain formula is given by,

$$T = \frac{\text{output variable}}{\text{input variable}} = \frac{\sum_K M_K \Delta_K}{\Delta}$$

where M_K is the K^{th} forward path gain, Δ is the determinant of the graph, given by,

$$\Delta = 1 - \sum P_{m1} + \sum P_{m2} \;\cdots\; (-1)^r \sum P_{mr}$$

where P_{mr} is the product of the gain of m^{th} possible combination of r non touching loops.

or $\Delta = 1 -$ (sum of gains of individual loops) + sum of gain products of possible combinations of 2 non touching loops) − (sum of gain products of all possible combinations of 3 nontouching loops) + ---

Δ_K : is the value of Δ for that part of the graph which is non touching with K^{th} forward path.